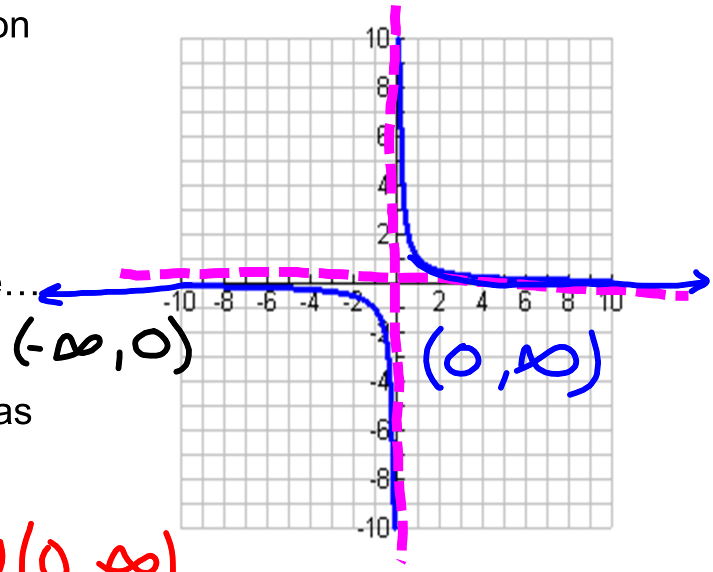
Parent Function

- The parent function is $\frac{1}{x}$
- The graph of the parent rational function looks like…



- The graph is not continuous and has asymptotes

Domain: * ℝ, $x \neq 0$ * $(-\infty,0)\cup(0,\infty)$

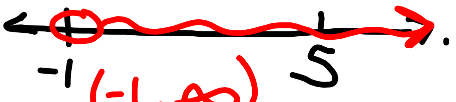
Range: $(-\infty,0)\cup(0,\infty)$

zeros: none

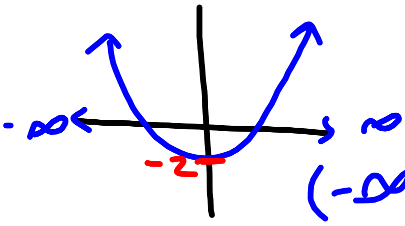
[redacted]

Interval Notation

(parentheses are used for ±∞ and to show the value is not on the graph



$(-1,\infty)$

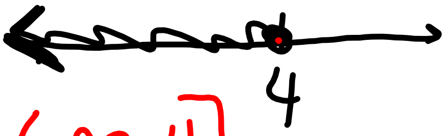


$(-\infty,\infty)$

all real #s

[Point is on the graph

Range $[-2,\infty)$



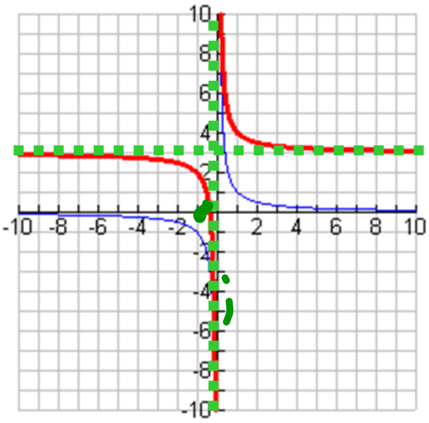
$(-\infty,4]$

Vertical Asymptote

- If (x – a) is a factor of the denominator of a rational function but not a factor of the numerator, then x = a is a vertical asymptote of the graph of the function.

discontinuous

$y = \frac{1}{x} + 3$



Domain:
$(-\infty, 0) \cup (0, \infty)$

Range:
$(-\infty, 3) \cup (3, \infty)$

Zeros: $(-\frac{1}{3}, 0)$

$0 = \frac{1}{x} + 3$

$x \cdot -3 = \frac{1}{x} \cdot x$

$-3x = 1$

$x = -\frac{1}{3}$

$$\frac{1}{(x+3)}$$

Domain:

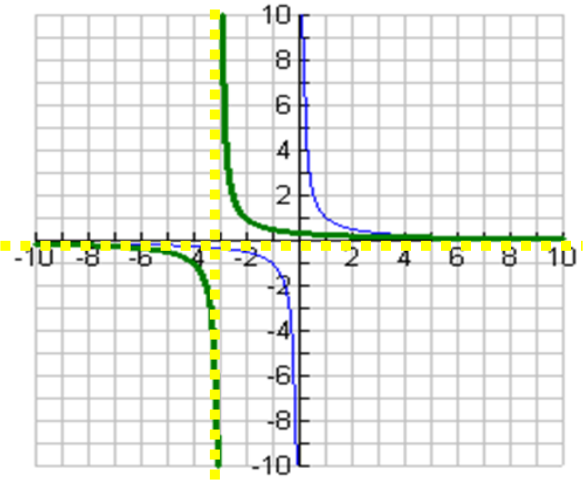
$(-\infty,-3)\cup(-3,\infty)$

Range:

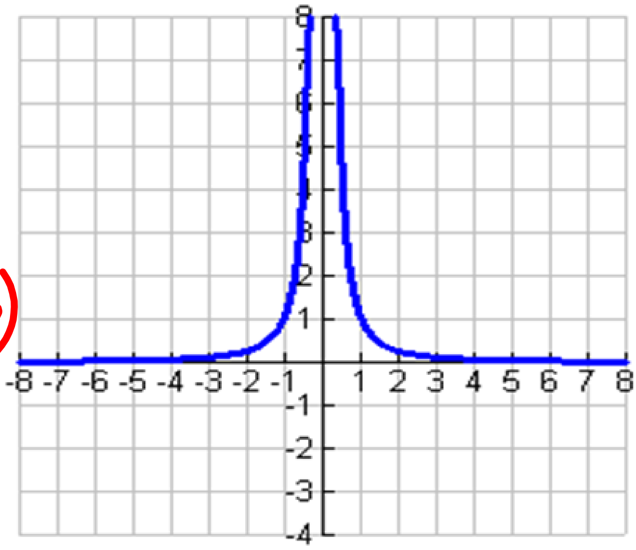
$(-\infty,0)\cup(0,\infty)$

Zeros:

none



$$\frac{1}{x^2}$$



Domain:

$(-\infty,0)\cup(0,\infty)$

Range:

$(0,\infty)$

Zeros:

none

$$f(x) = \frac{1}{(x+2)(x-3)}$$

where is this function undefined?

$x \neq -2, 3$

Vertical Asymptotes?

$x = -2$

$X = 3$

$$f(x)=\frac{\cancel{x-1}}{(x+4)(\cancel{x-1})}$$

Where is this function undefined?

$x \neq -4, 1$

Are they Vertical Asymptotes?

only $x = {}^{-}4$

Hole (in the graph)

- If (x – b) is a factor of both the numerator and denominator of a rational function, then there is a hole in the graph of the function where x = b.
- The exact point of the hole can be found by plugging b into the function after it has been simplified.

The numerator and denominator

must be in factored form

$$f(x)=\frac{x-3}{x^2+x-12}$$

1. factor numerator and denominator

$$\frac{\cancel{x-3}}{(x+4)\cancel{(x-3)}}$$

② Where is graph undef ?
$x \neq 3, -4$

③ Vertical Asymptote

VA: $x=-4$

④ Hole

hole: $x=3$ $(3, \frac{1}{7})$

⑤ Simplify the rational

Simplify: $\frac{1}{x+4}$

⑥ Plug the x for the hole into the simplified rational

$f(x)=\frac{1}{3+4}$

$\frac{1}{7}$

Identify vertical asymptotes & holes.

$$f(x)=\frac{3-2x-x^2}{x^2+x-2}$$

$$\frac{-(x^2+2x-3)}{x^2+x-2}$$

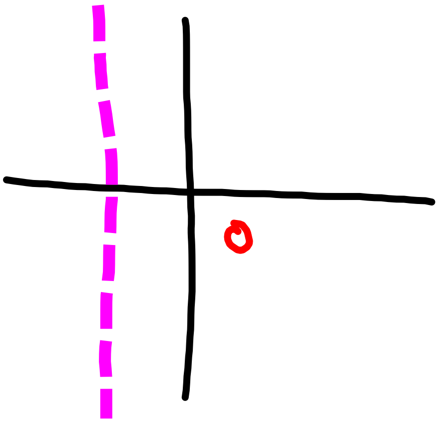
$$\frac{-(x+3)\cancel{(x-1)}}{(x+2)\cancel{(x-1)}}=\frac{-(x+3)}{x+2}$$

VA: $x=-2$

$$\frac{-4}{3}$$

hole: $x=1$

$(1, -\frac{4}{3})$



Identify the holes, VA, HA, and zeros. Sketch the graph and write the domain and range.

1) $f(x) = \dfrac{x^3 + x^2 - 2x}{-4x^2 - 12x}$

$$\frac{\cancel{x}(x^2+x-2)}{-4\cancel{x}(x+3)}$$

$$\frac{-2}{-12}$$

VA: $X=-3$

hole: $x=0$

$(0, \frac{1}{6})$

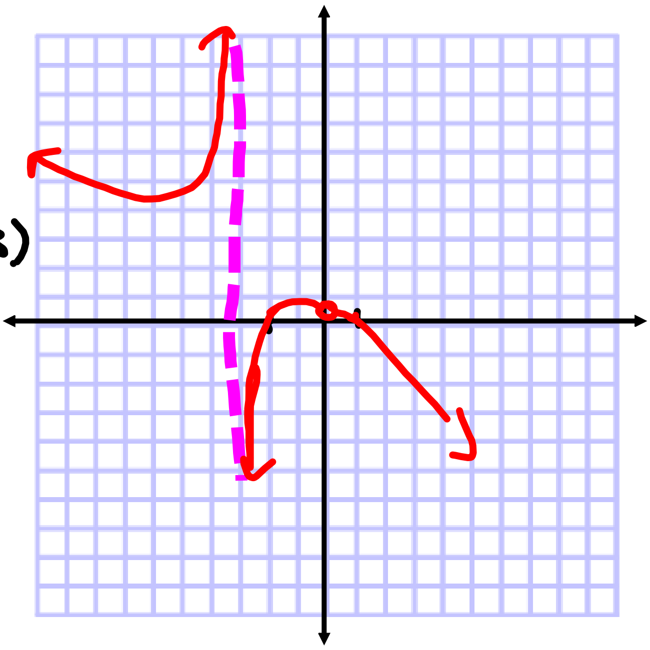
Zeros

$$0 = \frac{x^2+x-2}{-4(x+3)} \cdot -4(x+3)$$

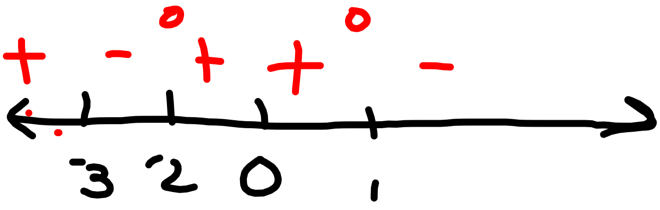
$0 = x^2+x-2$

$(x+2)(x-1)$

$x = -2, 1$



Sign Line



$$\frac{x^2+x-2}{-4(x+3)}$$

Horizontal Asymptotes

- **Degree of numerator = Degree of denominator**

  Horizontal Asymptote: $y = \frac{\text{coefficient of numerator}}{\text{coefficient of denominator}}$

- **Degree of numerator < Degree of denominator**

  Horizontal Asymptote: y=0

- **Degree of numerator > Degree of denominator**

  Horizontal Asymptote: None

$y = \frac{x^2+2}{3x^2-4}$

HA: $y = \frac{1}{3}$

$y = \frac{x^4-2}{x^3}$

$(x+1)(x^2-x+1)$

$$h(x) = \frac{x^3+1}{x^2-4}$$

$(x+2)(x-2)$

VA: $x=2, x=-2$

HA: no

hole: no

original problem $\frac{x^3+1}{x^2-4}$

oblique

Long division

$$\begin{array}{r|l} & \boxed{x} \\ x^2+0x-4 & x^3+0x^2+0x+1 \\ & -x^3 \mp 0x^2 + 4x \\ \hline & \qquad 4x+1 \end{array}$$

oblique $y=x$

Don't need the remainder

Zeros

$(x+1)(x^2-x+1)=0$

$(x+1)=0$ $\quad x^2-x+1=0$

$x=-1$ $\quad \frac{1 \pm \sqrt{1-4(1)(1)}}{2}$